# MEASURING PARTISAN BIAS IN ELECTORAL SYSTEMS WITH MULTIMEMBER DISTRICTS 

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This research is preliminary. Please quote.


#### Abstract

Partisan bias is said to exist when two parties with equal number of votes are apportioned different number of seats. It is thus a dyadic relation not to be confounded with deviation for any specific standard (e.g. large party positive bias, the typical deviation from proportionality). Partisan bias has been aptly studied for electoral systems with single member districts, whereas for multimember systems the literature has insisted on proportionality indices. We extend the idea of partisan bias to any kind of (single tier) electoral system. Bias is measured as the additive result of an efficiency component (related to the heterogeneity of districts), a vote-weight component (related mainly to malapportionment and turnout rates) and third party effects.


Partisan bias is the non neutral bias in the distribution of seats among parties after elections, favoring one party over another for reasons that cannot be readily justified by the total level of electoral support. It is a consequence of districting, although its causes are multiple, some of which are related to the behaviour of voters, and others to de design of the electoral system.

The term "bias" is normally used in connection to the majority bias, or large party bias, observable in every known electoral system. Exact proportional representation is used as a measuring rod for bias. Partisan bias, however, refers to the difference in treatment for two parties regardless their size. It is not neutral, because it is predicted to appear even if two parties were equal in the number of votes collected. Unlike majority bias, partisan bias cannot occur if the allocation of representatives takes place in a single district. Hence, districting is a necessary condition for bias. It is also almost sufficient, for in a districted electoral system the conditions for unbiased results are empirically unlikely to be satisfied (****).

Given that partisan bias is empirically pervasive, it is important to have adequate measures. The line between inevitable bias and systemic bias is difficult to draw. Obvious indicators are size and randomness. If bias is moderate and may affect one or another party without a clear temporal pattern, it is perhaps inevitable. If bias is strong and discriminates one party in some regular way, we might suspect of political intention, and a case for electoral reform might be established.

For the same reason, it is also important to be able to decompose the sources of bias. Not every source of bias is seen as equally illegitimate, if at all. For example, bias due to malapportionment has worse democratic credentials than bias due to local concentration of the vote; and even malapportionmet is often the consequence of well entrenched principles of territorial representation. If every form of bias were per se democratically intolerable, then we could only bear single district representation systems, which is clearly not the case.

The fact that partisan bias is impossible in single districts has no relation with proportional representation, a notion we associate with single (and large) districts. True, a pure proportional allocation is necessarily unbiased, but so is a presidential election by
majority, or a hypothetical single district in which the majority party holds the entire parliament, or some pre-established share (say $60 \%$ for the winner and $40 \%$ for the opposition). Empirically, it may be the case that politically viable unbiased systems for electing a legislative are associated to proportional representation. But the amelioration of partisan bias might entail partial reforms, like re-apportionment or re-districting, that keep the system within the pre-existing parameters as far as deviation from proportionality is concerned.

## CONCEPTUALIZATION AND MEASUREMENT STRATEGY

Partisan bias refers to the different treatment of two parties when two parties have the same number of votes. We shall define partisan bias as the difference between the fraction of the two-party seats and fraction of the two-party votes when both parties under study have $50 \%$ of the two-party vote, whatever the votes and seats of any other parties. Obviously, the value of bias for one party is the negative value for the other party. Some authors (e.g. Johnston et al ${ }^{* * *}$ ) prefer to use the actual number of seats as metric, rather than the fraction, but normalized figures have obvious advantages for comparative research.

We conceptualize partisan bias, as in most of the literature, as a dyadic relationship: the results may be biased for one party vis-à-vis a second party. If more than two parties are involved, partisan bias may be measured pairwise, and it might be positive with respect to one party and negative with respect to another. There are attempts in the literature to simultaneously measure bias in a three-party system with single districts, but it is not clear how they could be generalised to other kind of electoral systems or larger number of parties (****).

Since two-party equality is unlikely to be observed in practice, it can be approximated, in principle, either by statistical inference, adjusting a model to the observed repeated elections, or by some form of simulation. The second is our preferred route, for in repeated elections, even when sufficient data are available, the model for the seat-vote relationship in a given system may be difficult to ascertain. (Examples***)

Another alternative to simulations is to define bias as the difference between the actual fraction of seats and some normative expectation based on the actual fraction of the vote. This strategy is pursued by Gudgin and Taylor (***) who use the so-called "cube law" $(* * *)$ as a model for the partisan-unbiased result in a plurality system, and measure bias as the deviation from a prescription based on a parameterization of that law. This strategy is also implicit in the measurement of bias as the difference between seats and votes fractions, where proportional representation works as an ideal "model" from which empirical results deviate.

The obvious problem with this strategy is that, even if the cube law were the right model for plurality elections, we should need a true model for every specific electoral system for which neither that particular law nor pure proportional representation is an empirically valid expectation, that is, for almost any different electoral system.

Simulations are also problematic. The simplest idea, with a long tradition in electoral analysis, is to create an artificial electoral result by moving a constant percentage of the votes in every district so as the two parties have the same number of votes at the national level. This is normally termed "uniform swing" (****). Starting in the 90s, some authors have employed a far more sophisticated strategy, that obtains estimates for the bias parameter based on repeated simulations in which the swing is modelled in a more realistic fashion consistent with the data on electoral behavior (Gellman King, Jackman, Grofman et al***).

The latter strategy has the advantage that it provides estimated values with standard errors, through repeated simulation, of the bias parameter. It has the disadvantage of being very computing intensive. As for realism, it is always debatable what the more realistic counterfactuals are. For example, in the case of Spain, a realistic simulation should let the fraction of the vote of one of the two main parties vary very little (the Popular Party) and the other (the Socialist Party) vary a lot, with swingers mostly going to or from abstention. Yet, every existing proposal for the measurement of bias, including Grofman et al ( $* * *$ ) non uniform swing, assumes that participation remains the same in every simulated result, and hence it can be very unrealistic, no matter how many times repeated.

To the extent that we study one election at a time, realism is a less stringent concern. We shall estipulate that our measurement of partisan bias is "partisan bias assuming uniform swing". This should provide a useful indicator to compare elections and countries.
(*** One shot measure of partisan bias may lack robustness, in the sense that little perturbations could, under some conditions, increase or decrease the value of the indicator of bias in a significant way The indicator may be improved by making several measurements under different conditions: equality, several results near equality, reverse results....)

## Sources of partisan bias

Partisan bias appears in connection with a more efficient (in case or positive bias) or less efficient (for negative bias) distribution of the total number of votes across the districts, compared to a party with equal size, given the electoral system and the behaviour of electors, including abstainers and other party voters.

With regard to the institutional aspects of the electoral system, bias can be affected by the design of voting districts in several ways: malapportionment (unequal ratio of seats to electors), district boundaries, and electoral magnitude. All else being equal, the results are biased towards the party winning relatively more votes in over-represented constituencies. Again, the results are biased towards a party that wastes fewer votes, that is, the party that wins seats with votes closer to the minimal and farther to the maximal fraction of the vote that is associated with the number of seats obtained (Penadés***). In systems with constant electoral magnitude, the boundaries of the constituencies can render the distribution of the votes of a given party more or less efficient in terms of gaining seats, all else being equal. In systems with districts of varying number of seats, when two competing parties have the same total number of votes, the only way of minimizing wasted votes is getting seats in districts of smaller electoral magnitude than the rival party, all else being equal.

By way of example, figure *** shows how the number of necessary and sufficient votes increases with electoral magnitude when seats are allocated according to the D'Hondt formula.


Hence, besides malapportionment, there are two potential strategies for inducing partisan by institutional manipulation: first, given a (foreseen) vote distribution, district borders can be traced so as the distribution of one party is more efficient than the other -the infamous gerrymandering; second, if electoral magnitude is variable, districts can be traced so as a party wins in smaller districts and, hence, obtains a more favourable seats/votes ratio, all else being equal. (We could call them "districting" effects).

Less closely related to the design of the electoral system, at least arguably, are the effects on bias of abstainers and "other party" voters. Turnout affects bias in the same way malapportionment does, but it is not institutionalized. Voters in low-turnout constituencies become over-represented by virtue of the behaviour of non-voters. Otherparty competition has two potential consequences for partisan bias. Frist, by attracting voters, and detracting them from the two-party total of the parties under consideration, a third party acts on bias in a similar manner to abstainers or malapportionment: reducing, in some districts, the number of votes that are in fact contested in the two-party
competition. Second, by gaining seats, when this happens, it affects bias by altering the effective district magnitude, as far as the two parties under consideration are concerned (that is, the number of seats they divide between them). Under normal conditions both effects operate in opposite directions and, under proportional representation, tend to cancel each other, as we shall see $\left({ }^{* * *}\right)$.

There are also potential interactions among some of the effects mentioned that should be taken into account. Other party vote may interact with turnout and compensate its bias potential rather than reinforce it, or it may have different effects with different levels of malapportionment, all else being equal, and so on.

We do not study electoral magnitude and its variance separately. To do so, far more complex simulations on how voters would vote under different districting schemes would be necessary, and this exceeds the purpose of just measuring bias (reference***).

## THE ARITHMETIC OF PARTISAN BIAS IN MULTISEAT DISTRICTS

Let $n_{1 i}$ and $n_{2 i}$ be the number of votes obtained by parties 1 and 2 in district $i$, in an electoral system composed of $k$ districts, and let $s_{l i}$ and $s_{2 i}$ the number of seats obtained by parties 1 and 2 in district $i$. The total proportion of the two-party vote obtained by a given party is called $V$ :

$$
\begin{equation*}
V_{1}=\sum_{i} n_{1 i} / \sum_{i}\left(n_{1 i}+n_{2 i}\right) ; \tag{1}
\end{equation*}
$$

and the fraction of the two-party seats is called $S$ :

$$
S_{1}=\sum_{i} s_{1 i} / \sum_{i}\left(s_{1 i}+s_{2 i}\right) .
$$

We study bias as a two-party relationship. The rest is taken as parameters: the electoral system, the number of abstainers and the number of "third party" voters (including voters for any other option than the two parties under focus). Hence, we may write
$S=f(V)$, where the specific function depends on the electoral system and the vote distribution of the other parties. Obviously, $1-S=f(1-V)$.

Partisan bias is defined as
$B=f(0.5)-0,5$.

Define uniform swing
$v^{\prime} i=v i+-(0,5-V)$. such that $\sum v^{\prime} i=0{ }^{\prime}, 5$

For the purpose of decomposing the observed bias in measurable parts, it is natural to distinguish two main quantities. This will ease the calculations and their interpretation, by picking a natural landmark between the seats fraction and the vote fraction: the effective vote introduced bellow. In this paper we follow Taylor and Gudgin naming practice $\left({ }^{* * *)}\right.$ and call them "size component" and "distribution component", but we stress that they are not separable "effects", for "other parties" influence both.

The size component refers to the impact of the varying size of two-party vote totals in each district: how the local fractions aggregate into a national fraction. It comprises the direct effects of malapportionment, turnout, other-party votes (but not seats), and the interaction among them. The distribution component refers to the aggregation of the local seats-votes relation into the national seats-votes relation. In our conceptualization, it comprises the effect of other-party seats and the efficiency effect of vote distribution across districts.

After obtaining the relevant quantities, they are recombined to discern the analytically relevant effects: malapportionment, turnout, other parties and efficiency.

## The size component

To our knowledge, the first complete presentation of the arithmetic of the size component of the partisan bias for single-member districts is Gudgin and Taylor (****). We follow that formulation and generalize it to a system with varying district
magnitudes. We also clarify the equivalence, and slight differences, between the
 We have not been able to make full sense of the formulas given by Johnston et al (****), which is probably due to the persistence of errata even after correction ( ${ }^{* * *)}$ [Note: in the final published version there are terms first defined and then not used, which makes one strongly suspect of some kind of error]

We continue with some definitions and basic notation. Let $e_{i}$ be the number of electors in district $i$ and let $m_{i}$ the number of seats to be allocated in district $i$, with $\left(n_{1 i}+n_{2 i}\right) \leq$ $e_{i}$ and $\left(s_{l i}+s_{2 i}\right) \leq m_{i}$. For conceptual clarity it is useful to define also the number of electors per seat in a district as $a_{i}=e_{i} / m_{i}$ Now, let $t_{i}$ be the fraction of the electorate that voted -the turnout rate at district $i$; that is, given $h$ parties, and $h \geq 2, t_{i}=\left(n_{l i}+\right.$ $\left.n_{2 i} \ldots+n_{h i}\right) / e_{i}$. Next, let $p_{i}$ be the fraction of the turnout that voted for parties 1 and 2 in district $i$, that is $p_{i}=\left(n_{l i}+n_{2 i}\right) /\left(n_{l i}+n_{2 i} \ldots+n_{h i}\right)$. Finally, let $v_{l i}$ be the proportion of the two-party vote obtained by party 1 in district $i$, that is $v_{l i}=n_{l i} /\left(n_{l i}+n_{2 i}\right)$.

The aggregate two-party vote fraction (1) of the party under consideration can be rewritten as

$$
\begin{equation*}
V_{1}=\sum_{i} m_{i} a_{i} t_{i} p_{i} v_{1 i} / \sum_{i} m_{i} a_{i} t_{i} p_{i} \tag{2}
\end{equation*}
$$

The numerator is the total number of the votes received by the party at the national level as a function of the variables of interest; the denominator is the total number of twoparty voters.

This raw proportion of the two-party votes can be compared to the two-party "effective vote" proportion (a name used in Jackman and that we find adequate***), which is the hypothetical proportion of the vote obtained by the party if there were no malapportionment ( $a$ were constant across districts), the turnout rate were constant and so were the fraction of the votes accrued by parties 1 and 2 . If $a, t$, and $p$ are constants, then expression (2) is transformed into
$V_{1}^{*}=\sum_{i} m_{i} v_{1 i} / \sum_{i} m_{i}$.

This is the weighted average (by district magnitude) of the vote proportions of the party across all districts. It amounts to the fraction of the vote that the party averages per every seat in the election. In the special case in which the number of seats per district is a constant (single member districts or otherwise), this is just the average vote fraction across the $k$ districts

$$
V_{1}^{*}=\sum_{i} v_{1 i} / k
$$

If there were no differences in the absolute number of two-party voters other than those granted by the differences in number of seats contested, that is, if $\left(n_{1 i}+n_{2 i}\right) / m_{i}$ were a constant (or $n_{1 i}+n_{2 i}$ in the special case of fixed district magnitude), then the actual and the "effective" proportion would be identical. The size component of bias is then
$L=V^{*}-V$.

Clearly, in the simulations produced to measure partisan bias, $V$ is by definition equal to 0.5 and, hence,
$L=V^{*}-0.5$.

We may further decompose this segment of bias by comparing $V^{*}$ not to $V$ but to other hypothetical distributions. For instance, the direct malapportionment effect $(M)$ is given by the difference between $V^{*}$ and the hypothetical vote proportion if there were no turnout or third party effects ( $t$ and $p$ were constants) in this electoral system. That is, if only $m$ and $a$ are variables or, what amounts to the same, only $e$ is variable:

$$
M=V^{*}-\left(\sum_{i} m_{i} a_{i} v_{i} / \sum_{i} m_{i} a_{i}\right)=V^{*}-\left(\sum_{i} e_{i} v_{i} / \sum_{i} e_{i}\right) .
$$

Analogously, we can gather the direct effect of turnout ( $T$ ) by subtracting to the total vote the hypothetical vote proportion in which all but turnout were constant in the electoral system.
$T=V^{*}-\left(\sum_{i} m_{i} t_{i} v_{i} / \sum_{i} m_{i} t_{i}\right)$

In the special case of constant number of seats per district, this is
$T=V^{*}-\left(\sum_{i} t_{i} v_{i} / \sum_{i} t_{i}\right)$.

And the "other party" votes direct effect $(P)$ can be defined as

$$
P=V^{*}-\left(\sum_{i} m_{i} p_{i} v_{i} / \sum_{i} m_{i} p_{i}\right),
$$

or, in the special case of constant electoral magnitude, as

$$
P=V^{*}-\left(\sum_{i} p_{i} v_{i} / \sum_{i} p_{i}\right) .
$$

The arithmetic can be complicated by calculating the effects of two variables at a time, yet, it seems more sensible to group all interaction effects into one single residual term

$$
I=L+M+T+P,
$$

and the size component is

$$
L=M+T+P+I .
$$

But nothing prevents other decompositions that may be useful if, for example, we did not have separate turnout data, only census data and data on main parties:
$L=M+T \times P+I^{\prime}$.

More effects can be considered separately, by adding new variables in (2), like the separate fraction of different "third", "fourth" and so on parties, or by using those variables in different combinations, but the above factors will generally be more than sufficient.

Magnitude is left to vary in all the above expressions, when it does, for to hold it constant would alter a fundamental rule of the electoral system. The arithmetic relation of magnitude to vote totals should not be mistaken with the effect of variable $m$ in the seats-to-votes relationship, that is, in the efficiency of the votes.

## The distribution component

The distribution component $D$ of bias is defined as the difference between the two party fraction of the seats $(S)$ and the effective average of the two party vote $\left(V^{*}\right)$ when the raw fraction of the two party vote $(V)$ is set equal to 0,5 . If we write $S=f(V)$,

$$
D=f(0.5)-V^{*} .
$$

We shall distinguish two effects in this distribution component, the "shrinkage" $(H)$ of district magnitude by virtue of other parties obtaining part of the seats and the efficiency $(E)$ effect that arises from the fact that one party may waste more votes than the other. Hence, we write
$D=H+E$.

The shrinkage effect is the second effect of other party competition. We define the hypothetical vote fraction $V^{H}$ as a modification of $V^{*}$, in which the number of seats have been appropriately reduced, if necessary, as
$V_{1}^{H}=\sum_{i}\left(s_{1}+s_{2}\right) v_{1 i} / \sum_{i}\left(s_{1}+s_{2}\right)_{i}$,
and the shrinkage effect as

$$
H=V^{H}-V^{*} .
$$

It should be noticed that the use of $V^{*}$ instead of $V^{H}$ as a frontier between size and distribution components is a matter of convenience. First, it makes the interpretation of the index easier. Second unlike the separate factors considered in the decomposition of the size bias, the shrinkage effect might depend on the relative sizes of the two parties under study. Therefore, in the simulated results the shrinkage effect is liable to change with respect to the actual results, even with uniform swing. We prefer to consider part of the distribution component everything pertaining the simulated result. But there is nothing consequential in our choice.

The efficiency effect is the difference between the fraction of the seats received, when both parties have equal number of votes in the aggregate, and $V^{H}$, which is the average fraction of the votes per seat actually contested by the two parties:

$$
E=f(0.5)-V^{H}
$$

The $E$ component of bias shows the pure effect of districting. Even if malapportionment is absent and all else is constant, from participation to third party strength, a party may win considerably more (or less) seats than the other even with the same number of the votes, depending on the fortunate precise distribution of those votes. ***Word on gerrymandering
L (fixed)

## EXAMPLE...

Consider the following toy example of a four district system in which seats are allocated according to round quota and largest remainders (or similar formula) with ties solved in favour of the largest party. We calculate the partisan bias of this system comparing the results of parties one and two, which have the same number of votes.

| District | Seats <br> $(m)$ | Census <br> $(e)$ | Turnout <br> $(t)$ | Party 1 <br> $\left(n_{1}\right)$ | Party 2 <br> $\left(n_{2}\right)$ | Others <br> $(p)$ | P1 <br> $\left(s_{1}\right)$ | P2 <br> $\left(s_{2}\right)$ | Others |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $i$ | 3 | 3050 | 3000 | 2000 | 1000 | 0 | 2 | 1 | 0 |
| $j$ | 4 | 4050 | 4000 | 2000 | 2000 | 0 | 2 | 2 | 0 |
| $k$ | 1 | 1020 | 1000 | 0 | 500 | 500 | 0 | 1 | 0 |
| $l$ | 2 | 2500 | 2000 | 500 | 1000 | 500 | 0 | 1 | 1 |

Recall that $V$ and $S$ denote the total two-party vote and seats proportion, $V^{*}$ denotes the effective two party vote proportion (seats weighted average***), and $V^{H}$ the shrinkage corrected average in $\left({ }^{* * *}\right)$. Now, let us designate $V^{M}$ the malapportionment corrected average of expression $\left({ }^{* * * *} \quad V^{T}\right.$ the turnout corrected average of expression ${ }^{* * *}$ and $V^{P}$ the "other party vote" corrected average of expression ( ${ }^{* * *)}$.

|  | Milestones |  |
| :--- | :--- | :--- |
| $V$ | $\sum_{i} n_{1 i} / \sum_{i}\left(n_{1 i}+n_{2 i}\right)$ | 0.5000 |
| $V^{*}$ | $\sum_{i} m_{i} v_{1 i} / \sum_{i} m_{i}$ | 0.4667 |
| $V^{M}$ | $\sum_{i} e_{i} v_{i} / \sum_{i} e_{i}$ | 0.4572 |
| $V^{T}$ | $\sum_{i} m_{i} t_{i} v_{i} / \sum_{i} m_{i} t_{i}$ | 0.4756 |
| $V^{P}$ | $\sum_{i} m_{i} p_{i} v_{i} / \sum_{i} m_{i} p_{i}$ | 0.4782 |
| $V^{H}$ | $\sum_{i}\left(s_{1}+s_{2}\right) v_{1 i} / \sum_{i}\left(s_{1}+s_{2}\right)_{i}$ | 0.4815 |
| $S$ | $\sum_{i} s_{1 i} / \sum_{i}\left(s_{1 i}+s_{2 i}\right)$ | 0.4444 |


|  |  |  |
| :--- | :--- | :--- |
| Total bias | $S-V$ | -0.0556 |
|  | Main components |  |
| M | $V^{*}-V^{M}$ | +0.0095 |
| T | $V^{*}-V^{T}$ | -0.0090 |
| P | $V^{*}-V^{P}$ | -0.0115 |
| I | $V^{*}-V-M-T-P$ | -0.0224 |
| H | $V^{H}-V^{*}$ | +0.0148 |
| E | $S-V^{*}$ | -0.0370 |

The components of partisan bias show a few analytical interesting steps in the trajectory from votes to seats $S=V+M+T+P+I+H+E$. But giving the presence of an interaction term and two different effects of third parties, it is useful to read...

| Effects |  |  |
| :--- | :--- | :--- |
| Malapportionment effect | $M+I$ | -0.0129 |
| Turnout effect | $T+I$ | -0.0314 |
| Other party effect | $P+H+I$ | -0.0191 |
| Efficiency effect | $E$ | -0.0370 |

The inefficient distribution of votes has the most substantive impact on bias: it reduces the fraction of the seats of party 1 by an estimated 3.7 per cent points, all things being equal. The votes $(\mathrm{P})$ and seats $(\mathrm{H})$ distribution of third parties have opposite effects and nearly cancel out, but they have an impact on bias in conjunction with other inequalities (1.9 points). Differences in turnout have an estimated total effect of 3.1 negative points on the fraction of seats for party one, ceteris paribus, and malapportionment reduces that fraction by almost 1.3 points.

Grofman et al. $\left({ }^{* * *}\right)$ use a similar approach (see also Jackman ${ }^{* * *)}$ to the size effect, which they also define as the difference between the average proportion of the vote in the districts -the effective vote proportion- and the total proportion of the votes in the district. Within this effect, they use a weighting approach to differentiate malaportionment from other effects. Their approach is almost identical, but they do not take account of the interaction effects, which leads them to an imprecise claim about the measurement of the turnout and malapportionment effects.

They define $t_{i}$ as the weight of the turnout of the $i$-th district in total turnout
$u_{i}=\left(n_{1 i}+n_{2 i}\right) / \sum_{i}\left(n_{1 i}+n_{2 i}\right)$;
and define $d_{i}$ as the weight of the electorate of the $i$-th district in total electorate
$d_{i}=e_{i} / \sum_{i} e_{i} ;$
and, finally, define $w_{i}$ as the weight of the district magnitude of the $i$-th district in the total number of seats
$w_{i}=m_{i} / \sum_{i} m_{i}$,
which in their special case $(m=1)$ is a constant $w=1 / k$ for all $k$ districts.

Clearly, $V=\sum_{i} u_{i} v_{i}$, and $V^{*}=\sum_{i} w_{i} v_{i}$, and we can name $V^{M}$ (they use $M$ ) to the apportionment corrected average $V^{M}=\sum_{i} d_{i} v_{i}$.

Grofman et al. claim that the malapportionment effect is equal to $V^{*}-V^{M}$ and the turnout effect is equivalent to $V^{M}-V$, the difference between the census-weighted and participation-weighted average proportions of the vote. The second claim is exact -if there are no third party effects, which we may grant for the sake of the argument- for the
difference $V^{M}-V$ comprises not only the (direct) turnout effect but also the interaction of turnout and malapportionment (that we call $I$ ). However, $M$ does not exhaust the malapportionment effect if there is such interaction, for the total malapportionment effect is then $M+I$., the direct effect plus the (omitted) interaction. Again, if they meant "direct" malappportionment effect, then their claim on the turnout effect would not be exact.

## ILLUSTRATIVE APPLICATIONS OF THE PROCEDURES TO ESTIMATE THE DETERMINANTS OF PARTISAN BIAS IN A MULTIMEMBER DISTRICTS SYSTEM

Before addressing our main empirical findings, let us outline the main characteristics of the Spanish electoral system. According to the 1978 Constitution (art. 68), parliamentary elections in Spain are conducted under a closed-list proportional representation system in 50 multimember and 2 single-member districts. The method used to allocate the 350 seats is D'Hondt, and the average district magnitude is almost seven. In the 2008 elections, the district magnitude ranged from 1 in Ceuta and Melilla to 35 in Madrid, generating a huge imbalance in the size of the districts. As a consequence of the operation of the D'Hondt method and the comparatively low district magnitude, election outcomes are usually (but not only) very disproportional. For example, the Socialist Party won 48.3 per cent of the seats with only 43.9 per cent of the votes in 2008 (Table 1); but this pattern has constantly took place since 1977.

Tabla 1. Differences in the votes and seats shares in Spanish elections, 1977-2008 ${ }^{\text {a }}$

|  | Parties |  |  |  |  |  |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Elections | PCE/IU | PSOE | CDS | UPyD | UCD | AP/PP | CiU | PNV |
| 1977 | $-3,6$ | $+4,4$ | - | - | $+12,9$ | $-3,8$ | $-0,6$ | $+0,6$ |
| 1979 | $-4,2$ | $+4,1$ | - | - | $+12,9$ | $-3,5$ | $-0,5$ | $+0,4$ |
| 1982 | $-2,4$ | $+10,4$ | $-2,2$ | - | $-3,1$ | $+4,7$ | $-0,2$ | $+0,5$ |
| 1986 | $-2,7$ | $+8,5$ | $-3,8$ | - | - | $+3,9$ | $+0,1$ | $+0,2$ |
| 1989 | $-4,3$ | $+10,4$ | $-3,9$ | - | - | $+4,8$ | $+0,1$ | $+0,2$ |
| 1993 | $-4,5$ | $+6,0$ | - | - | - | $+5,5$ | 0,0 | $+0,2$ |
| 1996 | $-4,6$ | $+2,8$ | - | - | - | $+5,7$ | 0,0 | $+0,1$ |
| 2000 | $-3,2$ | $+1,6$ | - | - | - | $+7,8$ | $+0,1$ | $+0,5$ |
| 2004 | $-3,5$ | $+4,2$ | - | - | - | $+4,6$ | $-0,4$ | $+0,4$ |
| 2008 | $-3,2$ | $+4,6$ | - | $-0,9$ | - | $+3,6$ | $-0,1$ | $+0,5$ |

[^0]Table 2. Allocation of seats in 1977 and 2008 elections to the Spanish Parliament and simulations with changes in the vote shares of the two main parties ${ }^{\text {a }}$

|  | $\mathbf{1 9 7 7}$ <br> Elections | $\mathbf{1 9 7 7}$ <br> Simulated | $\mathbf{2 0 0 8}$ <br> Elections | $\mathbf{2 0 0 8}$ <br> Simulated |
| :--- | :---: | :---: | :---: | :---: |
| UCD | $\mathbf{1 6 6}(\mathbf{4 7 . 4})$ | $149(-4.8)$ | - | - |
| PSOE | $\mathbf{1 1 8}(\mathbf{3 3 . 7})$ | $135(+4.9)$ | $\mathbf{1 6 9 ( 4 8 , 3 )}$ | $165(-1,2)$ |
| PCE/IU | $\mathbf{1 9}(\mathbf{5 . 4})$ | $19(0.0)$ | $\mathbf{2 ( 0 , 6 )}$ | $2(0,0)$ |
| AP/PP | $\mathbf{1 6 ( 4 . 6 )}$ | $16(0.0)$ | $\mathbf{1 5 4 ( 4 4 , 0 )}$ | $158(+1,1)$ |
| PSP-US | $\mathbf{6 ( 1 . 7 )}$ | $6(0.0)$ | - | - |
| PDPC/CiU | $\mathbf{1 1 ( 3 . 1 )}$ | $11(0.0)$ | $\mathbf{1 0}(\mathbf{( 2 , 9 )}$ | $10(0,0)$ |
| PNV | $\mathbf{8 ( 2 . 3 )}$ | $8(0.0)$ | $\mathbf{6}(\mathbf{1 , 7 )}$ | $6(0,0)$ |
| UDC-IDCC | $\mathbf{2 ( 0 . 6 )}$ | $2(0.0)$ | - | - |
| EC-FED/ERC | $\mathbf{1 ( 0 . 3 )}$ | $1(0.0)$ | $\mathbf{3 ( 0 , 8 )}$ | $3(0,0)$ |
| EE | $\mathbf{1 ( 0 . 3 )}$ | $1(0.0)$ | - | - |
| CAIC | $\mathbf{1}(\mathbf{0 . 3})$ | $1(0.0)$ | - | - |
| INDEP | $\mathbf{1 ( 0 . 3 )}$ | $1(0.0)$ | - | - |
| UPyD | - | - | $\mathbf{1 ( 0 , 3 )}$ | $1(0,0)$ |
| BNG | - | - | $\mathbf{2 ( 0 , 6 )}$ | $2(0,0)$ |
| CC-PNC | - | - | $\mathbf{2 ( 0 , 6 )}$ | $2(0,0)$ |
| Na-Bai | - | - | $\mathbf{1 ( 0 , 3 )}$ | $1(0,0)$ |
| Total | $\mathbf{3 5 0}$ | $\mathbf{3 5 0}$ | $\mathbf{3 5 0}$ | $\mathbf{3 5 0}$ |

${ }^{\text {a }}$ Actual 1977 and 2008 electoral results are provided in bold in the first and third columns. In these columns, we also include in parentheses the seat shares obtained by each party. In the rest of columns, we include the number of seats that each party would obtain if UCD/PP and PSOE obtained an equal overall vote share, and in parentheses, the differences in percentages that would provoke.

Figure* Estimated bias with uniform swing, and its components, in Spain 1977-2011 (UCD in 1977-9 and PP otherwise are parties of reference)


Figure** Raw arithmetic components of bias without simulation in Spain 1977-2011 (UCD in 1977-9 and PP otherwise are parties of reference)


Figure *** Estimated Bias in selected countries


## DISCUSSION

Electoral systems are critical components of democracies. However, they can translate citizens' political preferences into seats in the legislature in a rather unfair way by benefitting one party over the others. Obviously, not all the types of electoral systems produce the same amount of bias. In fact, single-member districts plurality rule systems are widely known for provoking a considerable deviation from proportionality in the mapping of votes to seats that obtain each party. In this paper, we have presented a method to assess the various sources of partisan bias in multimember districts systems, and we have shown then its applicability by examining data from two general elections in Spain. The empirical results reported reveal that UCD, the main conservative party in the transition years, "out-biased" the Socialist Party in the first democratic elections. The overall amount of partisan bias in the Spanish electoral system, however, has generally diminished in magnitude over time. In this sense, the simulations performed using actual data from the 2008 elections (the last one celebrated so far) indicate that the pro-conservative bias that characterized the Spanish electoral rules during the first two decades seem to have disappeared and there is a Socialist advantage in that legislative body. Finally, we also see that the distribution component of emerges as the most important source of partisan bias.

Yet, we are aware of the fact that the findings presented in this article are not in and of them sufficient to provide a robust theory of the causes of partisan bias in electoral systems with multimember districts. For instance, it could be argued that 1977 and 2008 contests are far from archetypical and we should examine this phenomenon using data from other elections. In this sense, we plan to extend our analyses to the whole sequence of elections that have taken place in Spain since the democratic transition. Additionally, future comparative studies (perhaps, also including Portuguese elections) are necessary in order to address the potential lack of robustness of our results. ${ }^{1}$ Finally, what we have not completely identified is the exact role played by each of the potential sources of partisan bias. To sum up, this paper is still in its early stage since we need to collect and analyze additional information in order to figure out which causal mechanisms are at work. For this reason, we would appreciate any comments, suggestions and criticisms you may have on these, or any other issues you think we should be aware of.

REFERENCES [available upon request]

[^1]
[^0]:    ${ }^{\text {a }}$ Positive signs indicate situations of over-representation because parties obtain seats shares higher than their votes shares while negative signs entail situations of under-representation.

[^1]:    ${ }^{1}$ We would like to focus on Portugal because of the considerable similarities between it and Spain in terms of the electoral system.

